

Red Black Tree

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Why?

Consider the worse case scenario of BST insertion. If every new element n_i is larger all previous elements $n_1, n_2, \dots, n_{i-1}$, then the tree will degenerate into a linked list with height $\mathbb{O}(n)$.

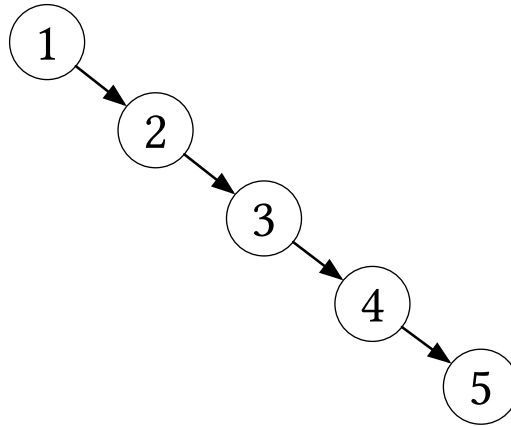


Figure 1: Degenerated BST

Solution: Balanced Tree

To handle those relatively rare cases, we can maintain the height of the tree to be $\mathcal{O}(\log n)$ through applying certain operations called rotations according to some rules.

This kind of tree is called a balanced tree. For heights of left and right subtrees of any node, h_l and h_r , $|h_l - h_r| \leq 1$.

While there's many types of balanced trees, we will focus on Red Black Trees today.

Red Black Tree Properties

A valid Red Black Tree satisfies the following properties:

1. Every node is either red or black
2. Empty leaf nodes are black
3. Direct children of a red node are black
4. On every path from the root to an empty leaf, the number of black nodes is the same

Example

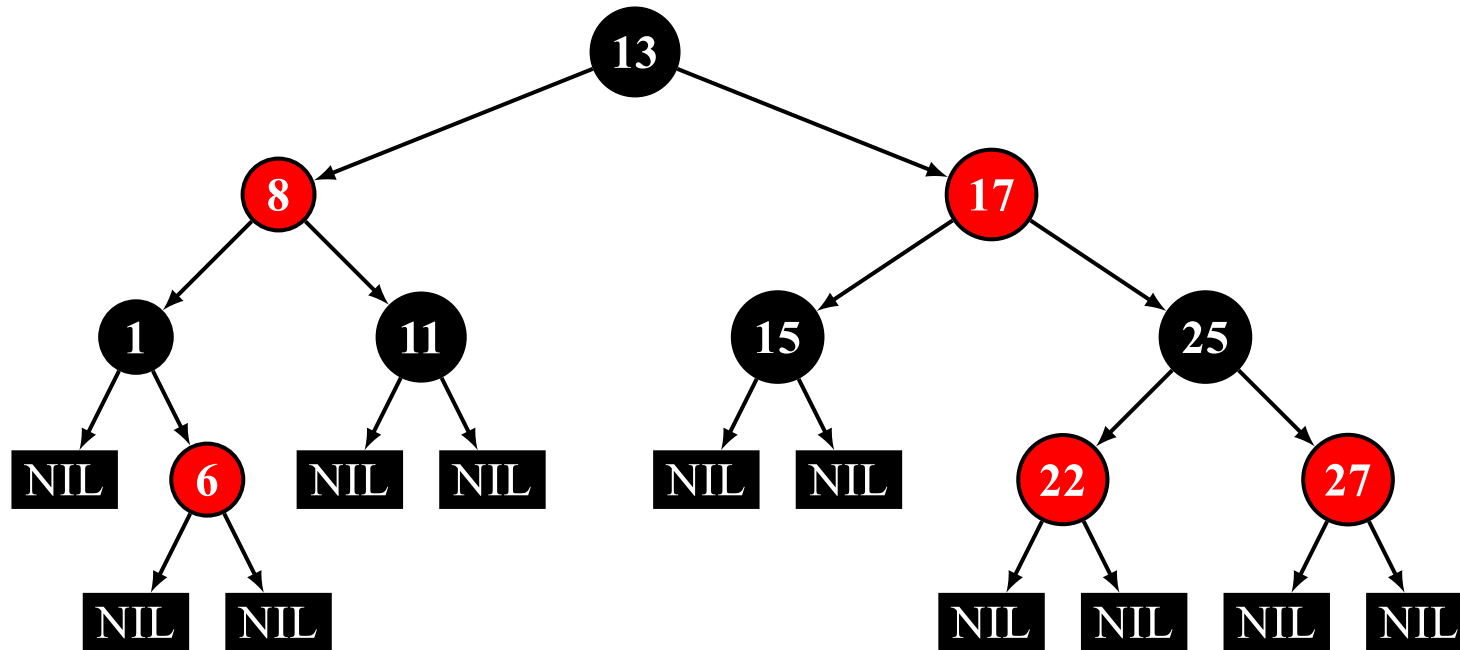


Figure 2: An example of a valid Red Black Tree [1]

How to maintain those properties?

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To maintain those properties, we use rotation operations.

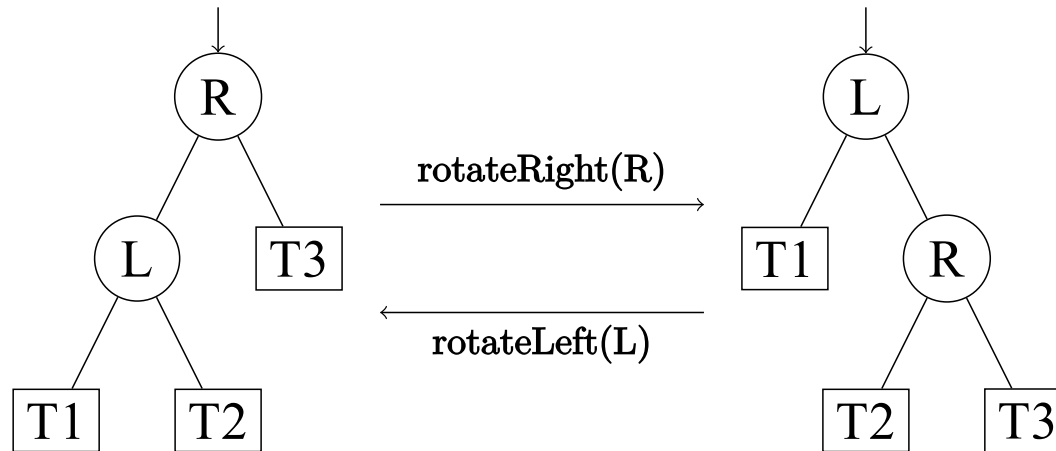


Figure 3: Left and right rotations [1]

Insertion: case #1

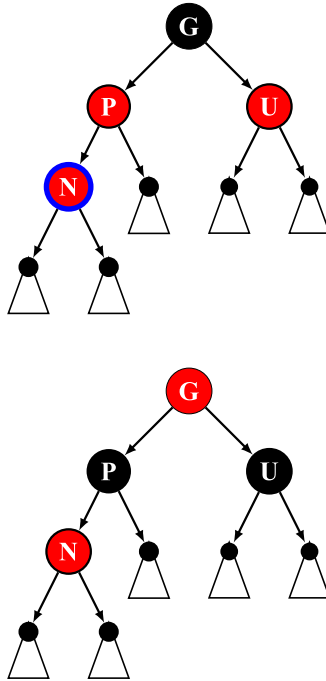


Figure 4: Insertion case #1 [1]

Insertion: case #2

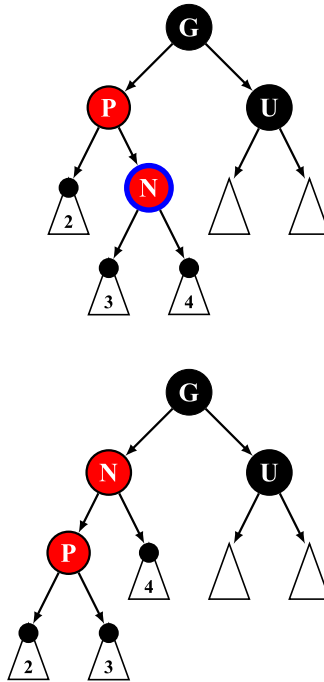


Figure 5: Insertion case #2 [1]

Insertion: case #3

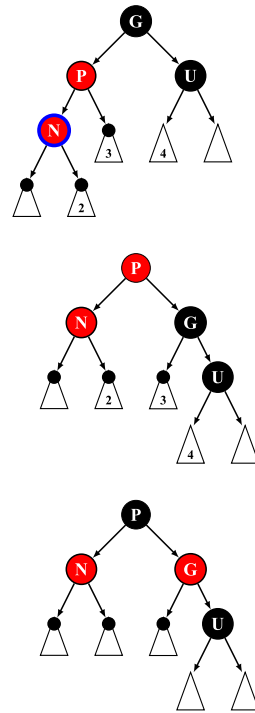


Figure 6: Insertion case #3 [1]

Deletion

Deletion is more complicated, but follows a similar idea. The several cases of deletion will be left as exercise.

Bibliography

- [1] yuhuoji, 0x03A. abc1763613206, auuuu4, c-forrest, C. C. E.-tainer, fanenr, happyZ. hsfzL. iamtwz, leverimmy, L. M. R. X. X. H. L. Y. Tiphereth-A trudbot, “Red Black Tree.” [Online]. Available: <https://oi-wiki.org/ds/rbtree/>