$$y'' + 3y' + 2y = e^x$$

$$\begin{aligned}
Y(0) &= C_1 \\
Y'(0) &= C_2 \\
\mathcal{L}[y] &= Y(s) \\
\mathcal{L}[y'] &= sY(s) - C_1 \\
\mathcal{L}[y''] &= s^2Y(s) - sC_1 - C_2 \\
\text{LHS} &= s^2Y(s) - sC_1 - C_2 + 3(sY(s) - C_1) + 2Y(s) \\
&= s^2Y(s) + 3sY(s) + 2Y(s) - sC_1 - C_2 - 3C_1 \\
\text{RHS} &= \mathcal{L}[e^x] = \frac{1}{s-1}
\end{aligned}$$

$$\begin{aligned}
s^2Y(s) + 3sY(s) + 2Y(s) - sC_1 - C_2 - 3C_1 &= \frac{1}{s-1} \\
Y(s)(s^2 + 3s + 2) &= \frac{1}{s-1} + sC_1 + C_2 + 3C_1 \\
Y(s) &= \frac{\frac{1}{s-1} + sC_1 + C_2 + 3C_1}{s^2 + 3s + 2} \\
Y(s) &= \frac{1 + (sC_1 + C_2 + 3C_1)(s-1)}{(s-1)(s+1)(s+2)} \\
Y(s) &= \frac{C_1s^2 + (2C_1 + C_2)s - (3C_1 + C_2 - 1)}{(s-1)(s+1)(s+2)}
\end{aligned}$$

$$\mathcal{L}^{-1}[Y(s)] = \frac{1}{6}(2 - 6C_1 - 6C_2)e^{-2t} + \frac{1}{2}(4C_1 + 2C_2 - 1)e^{-t} + \frac{1}{6}e^t$$