

$$\oint_C \mathbf{F} \cdot d\mathbf{r} \quad \text{where} \quad \mathbf{F} = \langle -3x^2 + 3yz + 4z^2, -x^2 + 3xz + 2y, 3xy - y^2 \rangle$$

$$\mathbf{r}(s, t) = \langle s, t, -3s - t + 5 \rangle \quad \{0 \leq s \leq 2, 0 \leq t \leq 1\}$$

$$\begin{aligned} \oint_C \mathbf{F} \cdot d\mathbf{s} &= \iint_S \nabla \times \mathbf{F} \cdot d\mathbf{S} \\ &= \int_0^1 \int_0^2 (\nabla \times \mathbf{F})(\mathbf{r}(s, t)) \cdot \left(\frac{\partial \mathbf{r}}{\partial s} \times \frac{\partial \mathbf{r}}{\partial t} \right) ds dt \\ &= \int_0^1 \int_0^2 \langle -2t, 8(-3s - t + 5), -2s \rangle \cdot \langle 3, 1, 1 \rangle ds dt \\ &= \int_0^1 \int_0^2 -6t - 24s - 8t + 40 - 2s ds dt \\ &= \int_0^1 \int_0^2 -14t - 26s + 40 ds dt \\ &= \int_0^1 -14ts - 13s^2 + 40s \Big|_{s=0}^2 dt \\ &= \int_0^1 -28t - 52 + 80 dt \\ &= -14t^2 - 28t \Big|_{t=0}^1 \\ &= 42 \end{aligned}$$