$$\iint_S \frac{x-y}{\sqrt{2z+1}}\,\mathrm{d}s \text{ where } S : \boldsymbol{r}(u,v) = \langle u+v, u-v, u^2+v^2\rangle \{0 \le u \le 1, 0 \le v \le 2\}$$

$$\frac{\partial \boldsymbol{r}}{\partial u} = \langle 1, 1, 2u\rangle, \quad \frac{\partial \boldsymbol{r}}{\partial v} = \langle 1, -1, 2v\rangle$$

$$\begin{aligned}
\iint_S \frac{x-y}{\sqrt{2z+1}}\,\mathrm{d}s &= \iint_D \frac{(u+v)-(u-v)}{\sqrt{2(u^2+v^2)+1}} \cdot \left\| \frac{\partial \boldsymbol{r}}{\partial u} \times \frac{\partial \boldsymbol{r}}{\partial v} \right\| \mathrm{d}u\,\mathrm{d}v \\
&= \iint_D \frac{(u+v)-(u-v)}{\sqrt{2(u^2+v^2)+1}} \cdot \|\langle 2u+2v, 2u-2v, -2\rangle\|\,\mathrm{d}u\,\mathrm{d}v \\
&= \iint_D \frac{(u+v)-(u-v)}{\sqrt{2(u^2+v^2)+1}} \cdot \sqrt{(2u+2v)^2+(2u-2v)^2+4}\,\mathrm{d}u\,\mathrm{d}v \\
&= \iint_D \frac{(u+v)-(u-v)}{\sqrt{2(u^2+v^2)+1}} \cdot 2\sqrt{2(u^2+v^2)+1}\,\mathrm{d}u\,\mathrm{d}v \\
&= \iint_D 4v\,\mathrm{d}u\,\mathrm{d}v \\
&= \int_0^2 \int_0^1 4v\,\mathrm{d}u\,\mathrm{d}v \\
&= \int_0^2 4v\,\mathrm{d}v \\
&= 2v^2\Big|_0^2 \\
&= 8
\end{aligned}$$