4B

$$
\begin{aligned}
\frac{\mathrm{d}V}{\mathrm{d}\theta} &= \frac{1}{\theta'(t)}\frac{\mathrm{d}}{\mathrm{d}t}X'(t) \\
&= \frac{1}{\theta'(t)}(f(t)\langle\cos\theta, \sin\theta\rangle + g(t)\langle-\sin\theta, \cos\theta\rangle) \\
&= \frac{f(t)}{\theta'(t)}\langle\cos\theta, \sin\theta\rangle \\
&= -\frac{GM}{r^2\theta'}\langle\cos\theta, \sin\theta\rangle \\
&= -\frac{GM}{C}\langle\cos\theta, \sin\theta\rangle \\
\int \mathrm{d}V &= -\frac{GM}{C}\int\langle\cos\theta, \sin\theta\rangle\,\mathrm{d}\theta \\
V &= \frac{GM}{C}\langle-\sin\theta, \cos\theta\rangle + \langle c_1, c_2\rangle
\end{aligned}
$$

When $c_1, c_2 = 0$:

$$
\begin{aligned}
r(t)\theta'(t) &= \frac{GM}{C} \\
-\frac{C}{r(t)} &= \frac{GM}{C} \\
r(t) &= \frac{C^2}{GM} \quad \text{(constant)}
\end{aligned}
$$

When $c_1, c_2 \neq 0$:

$$
\begin{aligned}
V &= X'(t) \\
\frac{GM}{C}\langle-\sin\theta, \cos\theta\rangle + \langle c_1, c_2\rangle &= r\theta'\langle-\sin\theta, \cos\theta\rangle \\
\frac{GM}{C}\langle-\sin\theta, \cos\theta\rangle + \langle c_1, c_2\rangle &= \frac{C}{r}\langle-\sin\theta, \cos\theta\rangle \\
\left(\frac{GM}{C}\langle-\sin\theta, \cos\theta\rangle + \langle c_1, c_2\rangle\right)\cdot\langle-\sin\theta, \cos\theta\rangle &= \frac{C}{r}\langle-\sin\theta, \cos\theta\rangle\cdot\langle-\sin\theta, \cos\theta\rangle \\
\frac{GM}{C} - c_1\sin\theta + c_2\cos\theta &= \frac{C}{r} \\
\frac{GM}{C^2} - \frac{c_1}{C}\sin\theta + \frac{c_2}{C}\cos\theta &= r^{-1} \\
C\left(\frac{GM}{C} - c_1\sin\theta + c_2\cos\theta\right)^{-1} &= r
\end{aligned}
$$

$$
\boxed{r = C\left(\frac{GM}{C} - c_1\sin\theta + c_2\cos\theta\right)^{-1}}
$$